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SU(3) mixing for excited mesons

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Abstract

The *SU*(3)-flavour symmetry breaking and the quark–antiquark annihilation mechanism are taken into account for describing the singlet–octet mixing for several nonets assigned by the Particle Data Group (PDG). This task is approached with the mass matrix formalism.

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1. Introduction

In the constituent quark model, the mesons are considered as bound states of a quark and an antiquark. Taking into account the *SU*(3)-flavour symmetry, the mesons are either in *SU*(3) singlets or octets: $\mathbf{3}\otimes\mathbf{3}=\mathbf{1}\oplus\mathbf{8}$. Nonetheless, due to the *SU*(3)-symmetry breaking, the isoscalar physical states appear as mixtures of the singlet and octet members. This singlet–octet mixing is also called *SU*(3) mixing. The inability of the Gell-Mann–Okubo mass formula [1] to adjust the masses of the pseudoscalar mesons has been considered as a suggestion for the inclusion of other effects such as the quark–antiquark annihilation into gluons. The failure of an *SU*(3)-invariant annihilation amplitude in attempting to solve the η–η' mass splitting [2, 3] led De Rujula *et al* [4] to propose that the quark–antiquark annihilation mechanism might not be *SU*(3)-invariant.

In a previous paper [5] the η–η' mass splitting was explained in an *SU*(3)-symmetry breaking framework. The physical states are mixtures of the isoscalar singlet and octet states and the amplitudes of quark–antiquark annihilation into gluons as well as the binding energies are supposed to be flavour dependent. Within this formulation, an extended expression for Schwinger's sum rule is satisfied. Also the *SU*(3) mixing angle obtained, $\theta=-19.51^\circ$, is consistent with the experimental data ($\theta\simeq-20^\circ$) from η and η' decays into pions [6]. As a very natural extension of the previous paper, we assume the *SU*(2)-symmetry breaking in the *SU*(3) mixing framework [7]. In this way, the pseudoscalar neutral mesons are mixtures of isoscalar and isovector states and the neutral pion participates in the mixing scheme. This model works well, but the result gives a hint that some significant effect possibly has not been considered. The strange result is that the ratio $m_s/m_u \simeq 2$ takes a somewhat large value, in comparison with those used in the constituent quark models ($m_s/m_u \simeq 1.3$–1.8). Our formulation is incompatible with fundamental models. If current quark masses were used the free parameters of the model would not be able to fit the masses of η and η'. In addition, the correct singlet–octet mixing angle would not be obtained.

The η–η' mixing scheme could be enlarged by the inclusion of gluonic degrees of freedom. The $\iota(1440)$ was interpreted as a strong glueball candidate due to its enhanced production in a gluon-rich channel [8]. The $\iota(1440)$, with the same quantum numbers as the η and η' system, motivated the study of the η–η'–ι mixing scheme [9–13]. Recently, the mass region near $\iota(1440)$ has been resolved into two states $\eta''(1410)$ and $\eta(1490)$ [14]. The first has been interpreted as being mainly a glueball mixed with $q\bar{q}$ and the second as mainly an $s\bar{s}$ radially excited state [15, 16]. Therefore, one is tempted to identify $\eta''(1410)$ as the remaining physical state in this extended mixing scheme [15–17] for ground states. On the other hand, the state $\eta(1490)$ is interpreted as a partner of the radially excited state $\eta(1295)$ [16]. The states $\eta(1295)$ and $\eta(1490)$ are the physical manifestations of mixtures among 2S excited states including solely light and strange quarks [15]. In a recent paper [18] we describe the η–η'–η'' and $\eta(1295)$–$\eta(1490)$ systems with the same formalism used in [5] but enlarging the mixing scheme to include glueballs. The small overlapping of the respective mass intervals suggests the possibility of mixing among ground states and radial excitations as considered by [19], however, in a first approximation, we assume that this 1S–2S mixing may be neglected. In searching for the best results of the branching ratios and of the decay widths involving the η, η' and η'' mesons, we have fixed all the parameters of the problem. This enlarged mixing scheme furnishes satisfactory results for the experimental data and improves the high value for the ratio m_s/m_u obtained in [18]. We obtained $m_s/m_u = 1.772$. Finally, we extend the mixing scheme to the excited states using the value of m_s/m_u determined for the ground state.

The nonets of axial (1^{++}, 1^3P_1) and tensor (2^{++}, 1^3P_2) mesons are well established [20]. The axial nonet consists of the isodoublet $K_{1A}(1340)$, the isovector $a_1(1260)$ and the isoscalars $f_1(1285)$ and $f_1(1510)$. The K_{1A} is a mixture of $K_1(1270)$ and $K_1(1470)$ with a close to 45° mixing angle [21]. The tensor nonet is formed by the isodoublet $K_2^*(1430)$, the isovector $a_2(1320)$ and the isoscalars $f_2(1270)$ and $f_2'(1525)$. Nonetheless, there are extra isoscalar states with quantum numbers and masses permitting them to be interpreted as partners of the nonets of axial and tensor mesons. The axial state $f_1(1420)$, observed in two experiments [22], has been considered by some authors [23] as a possible exotic candidate. On the other hand, there are two candidates for exotic tensor states: $f_2(1640)$ [24] and $f_J(1710)$ [25]. There is a controversy about the value of the spin of the $f_J(1710)$: it may be a scalar or a tensor state [26]. In another paper [27] we approached the problem of axial and tensor mesons where the candidates for exotics $f_1(1420)$ and $f_2(1640)$, or $f_2(1710)$, are supposed to be components of a quarkonia–gluonia mixing scheme similar to that previously applied to the pseudoscalar mesons [5]. In this last paper $m_s/m_u = 1.772$, determined in [18], has been used as an input. The predictions of the model for branching ratios and electromagnetic decays are incompatible with the experimental results. These facts suggest the absence of gluonic components in the axial and tensor isosinglet mesons analysed. On the other hand, the interpretations of the states $f_1(1420)$, $f_1(1510)$, $f_2(1640)$ and $f_J(1710)$ are controversial and, moreover, some of them need confirmation. The same mixing scheme was not applied to the scalar states because only the assignment for the scalar isodoublet is well established.

Here we analyse the mixing scheme for the nonets listed in table 13.2 of the Particle Data Group (PDG) [28] which have all the members suggested, including the scalar states, except the lowest pseudoscalar states (π, K, η, η'). To all intents and purposes, we ignore any quarkonia–gluonia interference. We also assume the $SU(2)$ invariance, which is justified by a preceding work [7] in which we have shown that the $SU(2)$-symmetry breaking is important to the mass splitting between the π^0 and $\pi^\pm$, but it has negligible effects in the η–η' mixing. We will suppose that the isospin symmetry breaking causes no mixing between the isoscalar members of the excited nonets.

2. The mass matrix formalism

Several kinds of mixing schemes have been proposed to give account of the peculiar properties of the isoscalar mesons. In some schemes, the physical states are written as linear combinations of pure quarkonia and gluonia states. The linear coefficients are generally related to the rotation angles and may be determined by the decay properties of, or into, the physical mesons [12, 13, 16, 17, 29, 30]. Another approach, in which the interference is considered at a more fundamental level, consists in writing a mass matrix for the physical states in the basis of the pure quarkonia and gluonia states. The elements of this mass matrix are obtained from a model that describes the process of interference. The mixtures of the basic states are induced by the off-diagonal elements. Thus, these elements must contain the amplitudes for transitions from one to another state of the basis. The eigenvalues of that matrix give the masses of the physical states and the corresponding eigenvectors give the proportion of quarkonia and gluonia in each meson [10, 15, 31].

In [5, 7, 18, 27] we have adopted a mixing scheme based on a mass matrix approach. The flavour-dependent annihilation amplitudes and binding energies are the mechanisms responsible for the quarkonia–gluonia mixing. Here a brief review of the mass matrix formalism used in previous papers is outlined only for the quarkonia mixing. The mass matrix in the basis $|u\bar{u}\rangle$, $|d\bar{d}\rangle$ and $|s\bar{s}\rangle$, including flavour-dependent binding energies and annihilation amplitudes, has matrix elements given by

$$\mathcal{M}_{ij} = (2m_i + E_{ij})\delta_{ij} + A_{ij} \tag{1}$$

where $i, j = u, d, s$. The contributions to the elements of the mass matrix are the rest masses of the quarks m_i, the eigenvalues E_{ij} of the Hamiltonian for the stationary bound state (ij) and the amplitudes A_{ij}, that account for the possibility of quarkonia–gluonia transitions. As in previous papers, we assume that E_{ij} and A_{ij} are not $SU(3)$-invariant quantities. Another basis also used consists of the isoscalar singlet and octet of the $SU(3)$,

$$|1\rangle = \frac{1}{\sqrt{3}}(\sqrt{2}|N\rangle + |S\rangle) \tag{2}$$

$$|8\rangle = \frac{1}{\sqrt{6}}(\sqrt{2}|N\rangle - 2|S\rangle) \tag{3}$$

where this basis is written in a form that presents a segregation of strange and nonstrange quarks,

$$|N\rangle = \frac{1}{\sqrt{2}}(|u\bar{u}\rangle + |d\bar{d}\rangle) \tag{4}$$

$$|S\rangle = |s\bar{s}\rangle. \tag{5}$$

Besides these states, we also need the isovector states

$$|\tilde{\pi}^0\rangle = \frac{1}{\sqrt{2}}(|u\bar{u}\rangle - |d\bar{d}\rangle). \tag{6}$$

In this basis, the mixing among the isoscalar and isovector states is caused by isospin symmetry breaking terms. Therefore, assuming the exact $SU(2)$-flavour symmetry, one need only consider the subspace spanned by the isoscalar states when the mass matrix reduces to a 2×2 matrix $\mathcal{M}_0$:

$$\mathcal{M}_0 = \begin{pmatrix} m_8 & m_{18} \\ m_{18} & m_1 \end{pmatrix} \tag{7}$$

where

$$m_1 = \tfrac{2}{3}(2m_u + m_s) + \tfrac{1}{3}(2E_{uu} + E_{ss}) + A_{11} \tag{8}$$

$$m_8 = \tfrac{2}{3}(m_u + 2m_s) + \tfrac{1}{3}(E_{uu} + 2E_{ss}) + A_{88} \tag{9}$$

$$m_{18} = \frac{2\sqrt{2}}{3}(m_u - m_s) + \frac{\sqrt{2}}{3}(E_{uu} - E_{ss}) + A_{18} \tag{10}$$

and

$$A_{88} = \tfrac{2}{3}(A_{uu} - 2A_{us} + A_{ss}) \tag{11}$$

$$A_{11} = \tfrac{1}{3}(4A_{uu} + 4A_{us} + A_{ss}) \tag{12}$$

$$A_{18} = \frac{\sqrt{2}}{3}(2A_{uu} - A_{us} - A_{ss}). \tag{13}$$

Using the mass relations for the isovector and isodoublet members,

$$M_1 = 2m_u + E_{uu} \tag{14}$$

$$M_{1/2} = m_u + m_s + E_{us} \tag{15}$$

where the annihilation effects are absent, only the rest masses of the quarks and the binding energies contribute to the physical masses. The notation uses subscripts to M to identify the isospin. Defining

$$M_{1/2}^{(\varepsilon)} = M_{1/2} + \varepsilon \tag{16}$$

where

$$\varepsilon = \frac{E_{uu} + E_{ss}}{2} - E_{us} \tag{17}$$

the elements of the mass matrix $\mathcal{M}_0$ are found to be

$$m_1 = \tfrac{1}{3}\big(2M_{1/2}^{(\varepsilon)} + M_1\big) + A_{11} \tag{18}$$

$$m_8 = \tfrac{1}{3}\big(4M_{1/2}^{(\varepsilon)} - M_1\big) + A_{88} \tag{19}$$

$$m_{18} = \frac{2\sqrt{2}}{3}\big(M_1 - M_{1/2}^{(\varepsilon)}\big) + A_{18}. \tag{20}$$

The above results show that the $SU(3)$-symmetry breaking gives rise to off-diagonal elements in the mass matrix. These elements are generated not only by the gluon annihilation amplitudes but also by influences due to the differences in the binding energies. These off-diagonal elements are responsible for the mixing effects among the states comprising the physical mesons. We adopt an expression for the amplitude of the process $q\bar{q} \leftrightarrow gg \leftrightarrow q'\bar{q}'$ similar to that of Cohen *et al* [32] and Isgur [33], where the numerator of the two-gluon annihilation amplitude expression is assumed to be an $SU(3)$-invariant parameter, which means that we parametrize the annihilation amplitude in the form

$$A_{qq'} = \frac{\Lambda}{m_q m_{q'}}. \tag{21}$$

The phenomenological parameter Λ is to be determined. Then, the amplitudes become

$$A_{11} = \tfrac{1}{2}(2 + r_1)^2 r_2 \tag{22}$$

$$A_{88} = \tfrac{2}{3}(1 - r_1)^2 r_2 \tag{23}$$

$$A_{18} = \frac{\sqrt{2}}{3}(2 + r_1)(1 - r_1) r_2 \tag{24}$$

where

$$\frac{1}{r_1} = \frac{m_s}{m_u} \tag{25}$$

$$r_2 = \frac{\Lambda}{m_u^2}. \tag{26}$$

The invariants of the mass matrix $\mathcal{M}_0$ under a unitary transformation give the following mass relations for the isoscalar physical states,

$$M + \widetilde{M} = \mathrm{Tr}(\mathcal{M}_0) \tag{27}$$

$$M \times \widetilde{M} = \det(\mathcal{M}_0) \tag{28}$$

where M and $\widetilde{M}$ are the eigenvalues of the mass matrix $\mathcal{M}_0$ (masses of the isoscalar physical states). Their corresponding eigenvectors are the physical states $|M\rangle$ and $|\widetilde{M}\rangle$ which are mixtures of $|1\rangle$ and $|8\rangle$,

$$|M\rangle = \cos(\theta)|8\rangle - \sin(\theta)|1\rangle \tag{29}$$

$$|\widetilde{M}\rangle = \sin(\theta)|8\rangle + \cos(\theta)|1\rangle \tag{30}$$

where the coefficients of the eigenvectors are written in terms of the singlet–octet mixing angle given by

$$\theta = \arctan\left(\frac{m_8 - M}{m_{18}}\right). \tag{31}$$

In terms of strange and nonstrange quarks, (29) and (30) can be written as

$$|M\rangle = X|N\rangle + Y|S\rangle \tag{32}$$

$$|\widetilde{M}\rangle = \widetilde{X}|N\rangle + \widetilde{Y}|S\rangle \tag{33}$$

where

$$X = \widetilde{Y} = \frac{\cos(\theta) - \sqrt{2}\sin(\theta)}{\sqrt{3}} \qquad Y = -\widetilde{X} = -\frac{\sqrt{2}\cos(\theta) + \sin(\theta)}{\sqrt{3}}. \tag{34}$$

Eliminating A_{11} from (27) and (28) we obtain the generalized Schwinger sum rule:

$$(M + \widetilde{M})\big(4M_{1/2}^{(\varepsilon)} - M_1\big) - 3M\widetilde{M} = 4\big[2M_{1/2}^{(\varepsilon)} - \big(1 - r_1^2\big)r_2\big]\big(M_{1/2}^{(\varepsilon)} - M_1\big) + 3M_1^2. \tag{35}$$

Table 1. $SU(3)$ mixing angles for excited nonets. As done by the PDG [28], the isosinglets (mostly octet) are listed first (underlined in table) and their per cent contents of strange quarks are also shown. The values presented for $|S\rangle$ and θ correspond to the range $m_s/m_u = 1.3$–1.8. The values for the $1\,^3\mathrm{P}_0$ nonet are found taking into account the central value for the mass of $f_0(1370)$.

$N^{2s+1}L_J$	J^{PC}	Nonet members	$\|S\rangle$ (%)	θ (deg)
$1\,^3\mathrm{S}_1$	1^{--}	ρ, $K^*(892)$, $\underline{\phi}$, ω	99.9–100	36.9–36.4
$1\,^1\mathrm{P}_1$	1^{+-}	$b_1(1235)$, K_{1B}, $\underline{h_1(1380)}$, $h_1(1170)$	98.0–98.9	−62.9–−60.8
$1\,^3\mathrm{P}_0$	0^{++}	$a_0(1450)$, $K_0^*(1430)$, $\underline{f_0(1370)}$, $f_0(1710)$	89.7–94.1	−73.4–−68.8
$1\,^3\mathrm{P}_1$	1^{++}	$a_1(1260)$, K_{1A}, $\underline{f_1(1285)}$, $f_1(1420)$	5.4–2.3	−41.3–−45.9
$1\,^3\mathrm{P}_2$	2^{++}	$a_2(1320)$, $K_2^*(1430)$, $\underline{f_2'(1525)}$, $f_2(1270)$	99.2–99.6	30.1–31.5
$1\,^1\mathrm{D}_2$	2^{-+}	$\pi_2(1670)$, $K_2(1770)$, $\underline{\eta_2(1870)}$, $\eta_2(1645)$	99.7–99.8	−59.8–−59.8
$1\,^3\mathrm{D}_3$	3^{--}	$\rho_3(1690)$, $K_3^*(1780)$, $\underline{\phi_3(1850)}$, $\omega_3(1670)$	99.5–99.8	31.4–32.4
$1\,^3\mathrm{F}_4$	4^{++}	$a_4(2040)$, $K_4^*(2045)$, $\underline{f_4(2050)}$, $f_4(2220)$	0.3–0.2	−51.5–−52.4
$2\,^1\mathrm{S}_0$	0^{-+}	$\pi(1300)$, $K(1460)$, $\underline{\eta(1440)}$, $\eta(1295)$	∼100–∼100	−55.4–−55.2
$2\,^3\mathrm{S}_1$	1^{--}	$\rho(1300)$, $K^*(1410)$, $\underline{\omega(1420)}$, $\phi(1680)$	10.3–3.9	54.0–46.7

To our knowledge, this generalized sum rule was obtained for the first time in [5]. Note that the ordinary Schwinger sum rule [2] can be recovered using $r_1 = 1$ in (35). Equations (27) and (28) can also be solved for r_1 and r_2 giving

$$\frac{m_s}{m_u} = \frac{\sqrt{2}}{2}\sqrt{\frac{(M - M_1)(\widetilde{M} - M_1)}{\left(\widetilde{M} + M_1 - 2M_{1/2}^{(\varepsilon)}\right)\left(2M_{1/2}^{(\varepsilon)} - M - M_1\right)}} \tag{36}$$

$$\frac{\Lambda}{m_u^2} = \frac{(\widetilde{M} - M_1)(M - M_1)}{4\left(M_{1/2}^{(\varepsilon)} - M_1\right)}. \tag{37}$$

The invariants of the mass matrix are functions of m_s/m_u, $\Lambda\big/m_u^2$ and ε. These quantities are not all free. Equations (27) and (28) impose some constraints among them. The equations are to be solved for $\Lambda\big/m_u^2$ and ε by considering m_s/m_u in a range of values consistent with those usually adopted when using constituent quark masses in a nonrelativistic quark model ($m_s/m_u = 1.3$–1.8). To find the solutions, one needs to solve a second degree algebraic equation. One of those solutions is an extraneous root and the criterion to get rid of it is comparison with the solution obtained for the $SU(3)$ mixing angle (31) in the case of $SU(3)$-invariant amplitudes and binding energies. Our choice consists of the mixing angle nearest to that $SU(3)$-invariant mixing angle.

3. Mixing in excited states

The mixing scheme briefly presented in the previous section, ignoring any quarkonia–gluonia mixing, is now applied to the excited members of the nonets. Attention will be paid to the referred assignments in table 13.2 of the PDG [28], even for the cases which are controversial. These results, corresponding to the range $m_s/m_u = 1.3$–1.8, are summarized in table 1.

3.1. $1\,^1S_0\ (0^{-+})$

The ground-state pseudoscalar nonet (π, K, η, η') has already been considered in [18], where an enlarged mixing scheme including gluonia is shown to be necessary. Putting the present mixing scheme to the test for this nonet without gluonic degrees of freedom ends in a complete fiasco in the range of m_s/m_u considered.

3.2. $1\,^3S_1\ (1^{--})$

The ground-state vector nonet (ρ, $K^*(892)$, ω, ϕ) has been well established for a long time. It presents an $SU(3)$ mixing angle near to ideal ω–ϕ. It can be found that ϕ presents 99.9–100% of strange quarks and mixing angles in the range 36.9–36.4°. These values are to be compared with that listed by PDG ($\theta = 36°$).

3.3. $1\,^1P_1\ (1^{+-})$

We found that the content of strange quarks in $h_1(1380)$ is much higher than in its isoscalar partner. This result is supported by the experimental data which show $h_1(1380) \to KK^*(892)+$ c.c. and $h_1(1170) \to \rho\pi$ being the only decay modes seen, at least up to now.

3.4. $1\,^3P_0\ (0^{++})$

For this nonet, we found that $f_0(1370)$ presents $89.7\%^{-18.5\%}_{+4.9\%}$–$94.1\%^{-10.9\%}_{+2.9\%}$ of strange quarks and $\theta = -73.4°^{-13.8°}_{+5.3°}$–$-68.8°^{-10.2°}_{+3.9°}$. These values were found taking into account that the broad resonance $f_0(1370)$ has mass equal to (1.35 ± 0.15) GeV. It is worthwhile remarking that among the two candidates for the $I = 1$ ($a_0(980)$, $a_0(1450)$) states and the four candidates for $I = 0$ ($f_0(400-1200)$, $f_0(980)$, $f_0(1370)$, $f_0(1710)$) acceptable results were found only for the isovector $a_0(1450)$ and for the isoscalars $f_0(1370)$ and $f_0(1710)$, namely the states listed in table 13.2 of the PDG. It should be highlighted, though, that $f_0(1710)$ contains only a small fraction of strange quarks in contrast to the indication of the PDG based on the naive quark model. In addition, it is observed that $f_0(1710)$ has a dominant $K\overline{K}$ decay mode and $f_0(1370)$ couples more strongly to $\pi\pi$ than to $K\overline{K}$.

3.5. $1\,^3P_1\ (1^{++})$

The $f_1(1420)$ competes for an $s\bar{s}$ assignment with percentages of 94.6–97.7% and mixing angles in the range -41.3–$-45.9°$ roughly in agreement with 75–84% and $\theta \sim -40°$ obtained by Close *et al* [30]. More recently, Li *et al* [34] obtained 92% of $s\bar{s}$ in $f_1(1420)$ and $\theta = -38.5°$. As a matter of fact, they obtained ∼50° and 51.5°, respectively, because they changed $|M\rangle$ for $|\widetilde{M}\rangle$, and vice versa, in (29), (30). The ratio of J/ψ radiative branching ratios into $f_1(1285)$ and $f_1(1420)$ and the ratio of the two-photon width of $f_1(1285)$ and $f_1(1420)$ are, using the formulae in [40], given by

$$\frac{\Gamma_{\gamma\gamma}(\widetilde{f}_1)}{\Gamma_{\gamma\gamma}(f_1)} = \left(\frac{5\widetilde{X} + \sqrt{2}\widetilde{Y}}{5X + \sqrt{2}Y}\right)^2 \left(\frac{\widetilde{M}}{M}\right)^3 \tag{38}$$

$$\frac{\Gamma_{\gamma\gamma}(\widetilde{f}_1)}{\Gamma_{\gamma\gamma^*}(f_1)} = \left(\frac{5\widetilde{X} + \sqrt{2}\widetilde{Y}}{5X + \sqrt{2}Y}\right)^2 \left(\frac{\widetilde{M}}{M}\right)^3 \tag{39}$$

$$\frac{B(J/\psi \to \gamma\widetilde{f}_1)}{B(J/\psi \to \gamma f_1)} = \left(\frac{\sqrt{2}\widetilde{X} + \widetilde{Y}}{\sqrt{2}X + Y}\right)^2 \left(\frac{\widetilde{P}}{P}\right)^3 \tag{40}$$

Table 2. Branching ratios and electromagnetic decay widths involving the axial mesons. f_1 and $\widetilde{f}_1$ stand for $f_1(1285)$ and $f_1(1420)$, respectively. The values presented in our model correspond to the range $m_s/m_u = 1.3$–1.8.

Observable	Our model	Experiment [28]
$\frac{\Gamma_{\gamma\gamma}(\widetilde{f}_1)}{\Gamma_{\gamma\gamma}(f_1)}$	0.43–0.29	$\frac{\Gamma_{\gamma\gamma}(\widetilde{f}_1)}{\Gamma_{\gamma\gamma}(f_1)} > \frac{1.4\pm0.8}{B(\widetilde{f}_1\to K\overline{K}\pi)}$
$\frac{\Gamma_{\gamma\gamma}(\widetilde{f}_1)}{\Gamma_{\gamma\gamma^*}(f_1)}$	0.43–0.29	$\frac{\Gamma_{\gamma\gamma}(\widetilde{f}_1)}{\Gamma_{\gamma\gamma}(f_1)} = \frac{0.63\pm0.34}{B(\widetilde{f}_1\to K\overline{K}\pi)}$
$\frac{B(J/\psi\to\gamma\widetilde{f}_1)}{B(J/\psi\to\gamma f_1)}$	1.11–0.81	$\frac{1.36\pm0.44}{B(\widetilde{f}_1\to K\overline{K}\pi)}$
$\frac{B(f_1\to\gamma\phi)}{B(f_1\to\gamma\rho)}$	0.005–0.002	0.013 ± 0.008

Table 3. Branching ratios involving the tensor mesons. f_2 and $\widetilde{f}_2$ stand for $f_2'(1525)$ and $f_2(1270)$, respectively. The values presented in our model correspond to the range $m_s/m_u = 1.3$–1.8.

Observable	Our model	Experiment [28]
$\frac{B(f_2\to\pi\pi)}{B(f_2\to K\overline{K})}$	0.024–0.012	0.0092 ± 0.0018
$\frac{B(\widetilde{f}_2\to\pi\pi)}{B(\widetilde{f}_2\to K\overline{K})}$	0.18–0.17	$0.055^{+0.005}_{-0.006}$
$\frac{B(J/\psi\to\gamma f_2)}{B(J/\psi\to\gamma\widetilde{f}_2)}$	0.25–0.28	0.34 ± 0.08

$$\frac{B(f_1 \to \gamma\phi)}{B(f_1 \to \gamma\rho)} = \frac{4}{9}\left(\frac{P_\phi}{P_\rho}\right)^3\left(\frac{X}{Y}\right)^2 \tag{41}$$

where f_1 and $\widetilde{f}_1$ stand for $f_1(1285)$ and $f_1(1420)$, respectively. Our results are summarized in table 2. In the table one can see that the ratios of (38), (40) and (39), (40) yield 0.39–0.36. On the experimental side, these ratios yield 1.03 ± 0.92 (an inferior limit) and 0.46 ± 0.40, respectively.

3.6. $1\,{}^3P_2\,(2^{++})$

For this nonet, we found mixing angles in the range 30.1–31.5° which are to be compared with the value 26° presented by the PDG and 27.5° found by Li *et al* [35]. The ratios of branching ratios, where f_2 and $\widetilde{f}_2$ stand for $f_2'(1525)$ and $f_2(1270)$, respectively, are given by

$$\frac{B(f_2 \to \pi\pi)}{B(f_2 \to K\overline{K})} = \frac{3X^2}{(\sqrt{2}Y + X)^2}\left(\frac{P_\pi}{P_K}\right)^5 \tag{42}$$

$$\frac{B(\widetilde{f}_2 \to K\overline{K})}{B(\widetilde{f}_2 \to \pi\pi)} = \frac{(\sqrt{2}\widetilde{Y} + \widetilde{X})^2}{3\widetilde{X}^2}\left(\frac{P_\pi}{P_K}\right)^5 \tag{43}$$

$$\frac{B(J/\psi \to \gamma f_2)}{B(J/\psi \to \gamma\widetilde{f}_2)} = \left(\frac{\sqrt{2}X + Y}{\sqrt{2}\widetilde{X} + \widetilde{Y}}\right)^2\left(\frac{P}{\widetilde{P}}\right)^3. \tag{44}$$

Our results and their comparison with the experimental data for this nonet are summarized in table 3.

3.7. $1\,^1D_2\,(2^{-+})$

We obtained values consistent with a near to ideal $\eta_2(1645)$–$\eta_2(1870)$ mixing and the second isoscalar being dominantly composed of $s\bar{s}$ as speculated by the PDG, although there are some expectations that it may be an hybrid [36, 37].

3.8. $1\,^3D_3\,(3^{--})$

For this nonet, we found mixing angles in the range 31.4–32.4° which are to be compared with the value 28° presented by the PDG.

3.9. $1\,^3F_4\,(4^{++})$

We found that $f_4(2220)$ is mainly an $s\bar{s}$ state. This result agrees with the suggestion of the PDG and has already been conjectured by Godfrey *et al* [38] and Blundell *et al* [39].

3.10. $2\,^1S_0\,(0^{-+})$

For the first radial excitation of the pseudoscalar nonet, we found that $\eta(1440)$ and $\eta(1295)$ present almost an ideal mixing with the first isoscalar being an $s\bar{s}$ state. Nevertheless, $\eta(1440)$ is now considered to be composed of two resonances: $\eta(1410)$ and $\eta(1490)$ [14]. The first one has been interpreted as being mostly a glueball mixed with $q\bar{q}$ and the second one as mostly an $s\bar{s}$ radially excited state [15, 16]. $\eta(1410)$ has been identified as the remaining physical state in the quarkonia–gluonia mixing scheme for the pseudoscalar ground states [15–18]. On the other hand, the state $\eta(1490)$ is interpreted as a partner of the radially excited state $\eta(1295)$ [15, 16, 18]. From this point of view, we found that $\eta(1490)$ is a $\sim$100% $s\bar{s}$ state and the mixing angle is in the range -55.4–$-55.2°$.

3.11. $2\,^3S_1\,(1^{--})$

The PDG proposes $\rho(1450)$ to be the isovector partner for this nonet, however we were unable to find consistent results even for the candidate $\rho(1700)$. On the other hand, the state $\rho(1300)$ reported by the LASS detector team [41], without any entry in the PDG tables, leads to almost satisfactory results. We found that $\phi(1680)$ has a sizeable $s\bar{s}$ component (89.7–96.1%), but is the $\omega(1420)$ which is mostly octet. This last result is in accord with the experimental data which show that $\phi(1680) \to KK^*(892) + \text{c.c.}$ is the dominant decay for $\phi(1680)$, and besides $\omega(1420)$ has no decay to $K\overline{K}$. It is worthwhile noting that the isoscalar $\omega(1420)$ is mostly octet instead of the $\phi(1680)$ state. The PDG suggests that the isodoublet $K^*(1410)$ could be replaced by the $K^*(1680)$ in this nonet. Unfortunately, with this replacement we are led to unsatisfactory results for all the ρ candidates.

4. Conclusion

In this paper, we have shown that a mixing flavour approach similar to that used to describe the isosinglet states of the pseudoscalar meson nonet [5] can also be used to describe isosinglet states for several angular momentum and radially excited nonets. In this approach, we assumed $SU(2)$ invariance. Moreover, we assumed that the constituent masses of the quarks, the binding energies of the states and the gluon annihilation amplitudes are not $SU(3)$-invariant quantities. The gluon annihilation amplitudes were parametrized according to the prescriptions of Cohen *et al* [32] and Isgur [33]. In addition to these assumptions, we disregarded the presence of gluonic components in the physical states. A linear 2×2 matrix formulation based on these assumptions was applied to seven orbitally excited nonets and two radially excited S-wave nonets.

The mixing scheme used in this paper works properly for the majority of the isoscalar states listed in table 13.2 of the PDG [28]. Ten nonets were analysed and eight of them appear to be compatible with the experimental predictions for their quark–antiquark content, branching ratios and radiative decays. Only in two cases do our results mismatch the experimental data. In these two cases, the isoscalar states are not well established. In the scalar sector, there are many resonances competing to be the isoscalar partners of this nonet. The mixing scheme only works using $a_0(1450)$, $f_0(1370)$ and $f_0(1710)$, the states listed in table 13.2 of PDG, nevertheless we found unsatisfactory results. The current status of the scalar nonet excludes any possibility of achieving a reliable conclusion. For the 2^3S_1 sector, a consistent result was reached using $\rho(1300)$, contrasting with the candidates listed by the PDG ($\rho(1450)$ and $\rho(1700)$). This point might be considered as a failure of our mixing scheme but the existence of two ρ states and maybe a third one ($\rho(1300)$) would suggest a non-trivial interpretation for this nonet.

To summarize, almost every nonet analysed in this paper can be satisfactorily described by our mixing scheme without any non-quark mesons. The relative success of this approach suggests that it might be used as a guide to the analyses of the quark–antiquark content of the physical mesons participating in a specific nonet.

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